Conjugation and conjugate symmetry

$$x(n) \longleftrightarrow X_k$$

$$y(n) = x^*(n) \longleftrightarrow Y_k = X^*_{-k}$$

The complex conjugate of a periodic signal $x(n)$ has the effect of complex conjugation and time reversal on the corresponding Fourier Series coefficients.

$$Y_k = \frac{1}{N} \sum_{n=N} y(n) e^{-jk\omega_0 n} = \frac{1}{N} \sum_{n=N} x^*(n) e^{-jk\omega_0 n}$$

$$= \underbrace{\left(\frac{1}{N} \sum_{n=N} x(n) e^{jk\omega_0 n}\right)}_{X_k} = \underbrace{\left(\frac{1}{N} \sum_{n=N} x(n) e^{-j(-k)\omega_0 n}\right)}_{X_{-k}}$$

$$= (X_{-k})^* = X^*_{-k}$$

Case (I) If $x(n)$ is real

If $x^*(n) = x(n)$

$$X^*_{-k} = X_k$$

$$X_{-k} = X^*_k .$$

Case (II) $X_{-k} = X^*_k = -X_k$

$x(n)$ real and odd

Case (III) even and odd.

$$x_e(n) = E\{x(n)\} \longleftrightarrow R_e\{X_k\}$$

$$x_e(n) = \frac{1}{2}[x(n) + x(-n)]$$

$$x_o(n) = \frac{1}{2}(x(n) - x(-n))$$

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11) Parseval's Theorem → Power

$x(n) \longleftrightarrow X_k$

$$\frac{1}{N} \sum_{n=N} |x(n)|^2 = \sum_{k=N} |X_k|^2$$

→ Average Power

It states that average power in a periodic signal equals the sum of the average power in all of its harmonic components.

$$\rightarrow \frac{1}{N} \sum_{n=N} |x(n)|^2 = \frac{1}{N} \sum_{n=N} x(n)\, x^*(n)$$

$$= \frac{1}{N} \sum_{n=N} x(n) \left(\sum_{n=N} X_k e^{jk\omega_0 n} \right)$$

$$= \frac{1}{N} \sum_{n=N} x(n) \left(\sum_{k=N} X^*(k)\, e^{-jk\omega_0 n} \right)$$

$$= \sum_{k=N} X^*_k \left(\frac{1}{N} \sum_{n=N} x(n)\, e^{-jk\omega_0 n} \right)$$

$$= \sum_{k=N} X^*_k X_k$$

$$= \sum_{k=N} |X_k|^2.$$

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